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Confinement potential in the dual monopole Nambu–Jona–Lasinio model with dual Dirac strings*

M. Faber[1], A.N. Ivanov[1,2], A. Müller[1], N.I. Troitskaya[1], M. Zach[1]

[1] Institut für Kernphysik, Technische Universität Wien, Wiedner Hauptstrasse 8-10, A-1040 Wien, Austria (faber, ivanov, mueller, zach@kph.tuwien.ac.at)
[2] State Technical University, Department of Nuclear Physics, 195251 St. Petersburg, Russian Federation

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Abstract. Interquark confinement potential is calculated in the dual monopole Nambu–Jona–Lasinio model with dual Dirac strings suggested in [2,3] as a functional of the dual Dirac string length. The calculation is carried out by explicit integration over quantum fluctuations of a dual-vector field (monopole–antimonopole collective excitation) around the Abrikosov flux line and string shape fluctuations. The contribution of the scalar field (monopole–antimonopole collective excitation) exchange is taken into account in the tree approximation because of the London limit regime. The dominant role of quantum fluctuations for the formation of the linearly rising part of the confinement potential is argued.

1 Introduction

The dual monopole Nambu–Jona–Lasinio model (MNJL) with dual Dirac strings as a continuum space-time analogy of compact quantum electrodynamics (CQED) [1] has been formulated in [2–4]. As has been shown in [1], CQED possesses the same nonperturbative phenomena as in low-energy QCD. The MNJL model is based on a Lagrangian that is invariant under magnetic U(1) symmetry, with massless magnetic monopoles self-coupled through a local four-monopole interaction [2,3]:

$$\begin{aligned}\mathcal{L}(x) = \bar{\chi}(x)\mathrm{i}\gamma^{\mu}\partial_{\mu}\chi(x) + G[\bar{\chi}(x)\chi(x)]^2\\ -G_1[\bar{\chi}(x)\gamma_{\mu}\chi(x)][\bar{\chi}(x)\gamma^{\mu}\chi(x)],\end{aligned} \tag{1.1}$$

where $\chi(x)$ is a massless magnetic monopole field, and G and G_1 are positive phenomenological constants. Below we will show that we have to choose $G_1 = G/4$ for the self-consistency of the theory in the one-loop approximation.

The magnetic monopole condensation accompanies the creation of monopole–antimonopole ($\bar{\chi}\chi$) collective excitations with the quantum numbers of a scalar Higgs meson field ρ and a dual-vector field C_{μ}.

For the derivation of an effective Lagrangian, the ρ and C_{μ} fields are introduced as cyclic variables.

$$\mathcal{L}(x) = \overline{\chi}(x)\mathrm{i}\gamma^{\mu}\partial_{\mu}\chi(x) - \mathcal{V}(x), \tag{1.2}$$

where $\mathcal{V}(x)$ is defined

$$-\mathcal{V}(x) = \overline{\chi}(x)\left(-g\gamma^{\mu}C_{\mu}(x) - \kappa\rho(x)\right)\chi(x) - \frac{\kappa^2}{4G}\rho^2(x) + \frac{g^2}{4G_1}C_{\mu}(x)C^{\mu}(x). \tag{1.3}$$

Now we can show that the vacuum expectation value (VEV) of the ρ field does not vanish. Toward this aim, we have to derive the equation of motion of the ρ field by varying the Lagrangian (1.1) with respect to the ρ field:

$$\frac{\partial\mathcal{L}(x)}{\partial\rho(x)} = -\kappa\overline{\chi}(x)\chi(x) - \frac{\kappa^2}{4G}\rho(x) = 0. \tag{1.4}$$

This leads to

$$\rho(x) = -\frac{2G}{\kappa}\overline{\chi}(x)\chi(x). \tag{1.5}$$

Taking the VEV of both sides of (1.5), we get

$$\langle\rho(x)\rangle = -\frac{2G}{\kappa}\langle\overline{\chi}(x)\chi(x)\rangle = -\frac{2G}{\kappa}\langle\overline{\chi}(0)\chi(0)\rangle, \tag{1.6}$$

where $\langle\overline{\chi}(0)\chi(0)\rangle$ is the magnetic monopole condensate. Thus, the nonzero value of the VEV of the ρ field is related to the monopole condensation. In order to deal with a physical scalar field, which we will call σ, we have to subtract $\langle\rho(x)\rangle$, i.e., $\sigma(x) = \rho(x) - \langle\rho(x)\rangle$. It is convenient to denote $\langle\rho(x)\rangle = M/\kappa$, where M is proportional to $\langle\overline{\chi}(0)\chi(0)\rangle$,

$$M = -2G\langle\overline{\chi}(0)\chi(0)\rangle, \tag{1.7}$$

and as will be shown below, M has the meaning of the magnetic monopole mass in the superconducting phase. (1.7) is the so-called gap equation, which testifies to the appearance of the nonzero mass of the magnetic monopoles

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in the superconducting phase, and leads to the suppression of direct transitions between the physical scalar field and the vacuum.

In terms of the σ field, the Lagrangian (1.2) reads

$$\mathcal{L}(x) = \overline{\chi}(x)\,(\mathrm{i}\gamma^\mu\partial_\mu - M)\,\chi(x) - \tilde{\mathcal{V}}(x), \qquad (1.8)$$

where now $\tilde{\mathcal{V}}(x)$ reads

$$-\tilde{\mathcal{V}}(x) = \overline{\chi}(x)\,(-g\gamma^\mu C_\mu(x) - \kappa\sigma(x))\,\chi(x) - \frac{\kappa^2}{4G}\rho^2(x) + \frac{g^2}{4G_1}C_\mu(x)C^\mu(x). \qquad (1.9)$$

Integrating over the magnetic monopole fields $\bar{\chi}(x)$ and $\chi(x)$, we arrive at the effective Lagrangian

$$\mathcal{L}_{\rm eff}(x) = \tilde{\mathcal{L}}_{\rm eff} - \frac{\kappa^2}{4G}\rho^2(x) + \frac{g^2}{4G_1}C_\mu(x)C^\mu(x), \qquad (1.10)$$

with $\tilde{\mathcal{L}}(x)_{\rm eff}$ defined as

$$\tilde{\mathcal{L}}_{\rm eff}(x) \;=\; -\,\mathrm{i}\left\langle\, x\left|\ln\frac{\mathrm{Det}(\mathrm{i}\,\hat{\partial}\,-\,M\,+\,\Phi)}{\mathrm{Det}(\mathrm{i}\,\hat{\partial}\,-\,M)}\right|x\,\right\rangle. \qquad (1.11)$$

Here we have denoted $\Phi = -g\gamma^\mu C_\mu - \kappa\sigma$, and $\sigma = \rho - M/\kappa$.

The effective Lagrangian $\tilde{\mathcal{L}}_{\rm eff}(x)$ can be represented by an infinite series

$$\tilde{\mathcal{L}}_{\rm eff}(x) = \sum_{n=1}^{\infty}\frac{\mathrm{i}}{n}\,\mathrm{Tr}_{\rm L}\left\langle x\left|\left(\frac{1}{M\,-\,\mathrm{i}\,\hat{\partial}}\,\Phi\right)^{n}\right|x\right\rangle = \sum_{n=1}^{\infty}\tilde{\mathcal{L}}^{(n)}_{\rm eff}(x)\,. \qquad (1.12)$$

The index L means the evaluation of the trace over the Lorentz indices. The effective Lagrangian $\tilde{\mathcal{L}}^{(n)}_{\rm eff}(x)$ is given by

$$\tilde{\mathcal{L}}^{(n)}_{\rm eff}\,(x) = \int\prod_{\ell=1}^{n-1}\frac{\mathrm{d}^4x_\ell\,\mathrm{d}^4k_\ell}{(2\,\pi)^4}\,\mathrm{e}^{-\,\mathrm{i}\,k_1\cdot x_1\,-\,\ldots\,-\,\mathrm{i}\,k_n\cdot x} \times\left(-\,\frac{1}{n}\,\frac{1}{16\,\pi^2}\right)\int\frac{\mathrm{d}^4k}{\pi^2\mathrm{i}}\mathrm{Tr}_{\rm L}\left\{\frac{1}{M\,-\,\hat{k}}\,\Phi\,(x_1) \times\frac{1}{M\,-\,\hat{k}\,-\,\hat{k}_1}\,\Phi\,(x_2)\,\ldots\Phi\,(x_{n-1}) \times\frac{1}{M\,-\,\hat{k}\,-\,\hat{k}_1\,-\ldots-\,\hat{k}_{n-1}}\,\Phi\,(x)\right\}. \qquad (1.13)$$

at $k_1 + k_2 + \ldots + k_n \,=\, 0$. The r.h.s. of (1.13) describes the one-massive-monopole-loop diagram with n vertices. The monopole-loop diagrams with two vertices $(n\,=\,2)$ determine the kinetic term of the σ field and give the contribution to the kinetic term of the C_μ field, while the diagrams with $(n \geq 3)$ describe the vertices of interactions of the σ and the C_μ fields. In accordance with the prescription given in [2,3], the effective Lagrangian $\tilde{\mathcal{L}}_{\rm eff}\,(x)$ should be defined by the set of divergent one-massive-monopole-loop diagrams with $n\,=\,1,2,3$ and 4 vertices. The evaluation of these diagrams gives

$$\begin{aligned}\mathcal{L}_{\rm eff}\,(x) &= \frac{1}{2}\,\frac{\kappa^2}{8\,\pi^2}\,J_2(M)\,\partial_\mu\,\sigma(x)\,\partial^\mu\,\sigma(x)\\ &-M\left[\frac{\kappa}{2\,G}\,-\,\frac{\kappa}{4\,\pi^2}\,J_1(M)\right]\sigma(x)\,+\\ &+\frac{1}{2}\left[-\,\frac{\kappa^2}{2\,G}\,+\,\frac{\kappa^2}{4\,\pi^2}J_1(M)\,-\,4\,M^2\,\frac{\kappa^2}{8\,\pi^2}\,J_2(M)\right]\sigma^2(x)\\ &-2\,M\,\kappa\,\frac{\kappa^2}{8\,\pi^2}\,J_2(M)\,\sigma^3(x)\,-\,\frac{1}{2}\,\kappa^2\,\frac{\kappa^2}{8\,\pi^2}\,J_2(M)\,\sigma^4(x)\\ &-\frac{g^2}{48\,\pi^2}\,J_2(M)\,\mathrm{d}C_{\mu\nu}(x)\,\mathrm{d}C(x)^{\mu\nu}\\ &+\left[\frac{g^2}{4G_1}\,-\,\frac{g^2}{16\,\pi^2}\,[J_1(M)\,+\,M^2\,J_2(M)]\right]\\ &\times C_\mu(x)\,C^\mu(x)\,,\end{aligned} \qquad (1.14)$$

where we have defined $dC^{\mu\nu}(x) \,=\, \partial^\mu C^\nu(x) \,-\, \partial^\nu C^\mu(x)$. Then, $J_1(M)$ and $J_2(M)$ are the following quadratically and logarithmically divergent integrals:

$$\begin{aligned}J_1(M) &= \int\frac{\mathrm{d}^4k}{\pi^2\,\mathrm{i}}\,\frac{1}{(M^2\,-\,k^2)} = \Lambda^2 - M^2\ln\left(1 + \frac{\Lambda^2}{M^2}\right) \,-\, \frac{\Lambda^2}{M^2\,+\,\Lambda^2},\\ J_2(M) &= \int\frac{\mathrm{d}^4k}{\pi^2\,\mathrm{i}}\,\frac{1}{(M^2\,-\,k^2)^2} = \ln\left(1 + \frac{\Lambda^2}{M^2}\right) \,-\, \frac{\Lambda^2}{M^2\,+\,\Lambda^2}.\end{aligned} \qquad (1.15)$$

In order to get the correct factors of the σ and C_μ field kinetic terms, we have to set [2,3]

$$\frac{g^2}{12\,\pi^2}\,J_2(M) \,=\, 1\;, \qquad \frac{\kappa^2}{8\,\pi^2}\,J_2(M) \,=\, 1\,. \qquad (1.16)$$

So the coupling constants are connected by the relation $\kappa^2 = 2g^2/3$ [2,3].

The effective Lagrangian (1.14) contains a term that is linear in the σ field. This part of the effective Lagrangian leads to direct transitions $\sigma \to$ vacuum. In the case of a physical σ field, such transitions should be suppressed. In order to suppress these transitions, we have to impose the constraint

$$\frac{1}{G} - \frac{1}{2\pi^2}\,J_1(M) = 0, \qquad (1.17)$$

where $J_1(M)$ can be connected with the monopole condensate via

$$\langle\bar{\chi}(0)\chi(0)\rangle = -\frac{1}{4\pi^2}MJ_1(M). \qquad (1.18)$$

Inserting (1.18) into (1.17), we arrive at the gap equation (1.7)

The coefficient in front of the last term in (1.14) defines the mass of the C_μ field:

$$M_C^2 = \frac{g^2}{2G_1} - \frac{g^2}{8\,\pi^2}\,[J_1(M) + M^2\,J_2(M)] \quad (1.19)$$

Picking up the gap equation (1.7) and the constraint (1.16), we bring the effective Lagrangian (1.14) to the form

$$\begin{aligned}\mathcal{L}_{\rm eff}(x) &= -\frac{1}{4}\,{\rm d}C_{\mu\nu}(x)\,{\rm d}C^{\mu\nu}(x) + \frac{1}{2}\,M_C^2\,C_\mu(x)\,C^\mu(x)\\ &+\frac{1}{2}\partial_\mu\,\sigma(x)\,\partial^\mu\sigma(x) - \frac{1}{2}M_\sigma^2\sigma^2(x)\left[1+\kappa\frac{\sigma(x)}{M_\sigma}\right]^2\\ &= -\frac{1}{4}\,{\rm d}C_{\mu\nu}(x)\,{\rm d}C^{\mu\nu}(x) + \frac{1}{2}\,M_C^2\,C_\mu(x)\,C^\mu(x)\\ &+\frac{1}{2}\partial_\mu\,\sigma(x)\,\partial^\mu\sigma(x) - \frac{1}{2}M_\sigma^2\sigma^2(x) + \mathcal{L}_{\rm int}[\sigma(x)], \quad (1.20)\end{aligned}$$

where $M_\sigma = 2\,M$ is the mass of the σ field, and $\mathcal{L}_{\rm int}[\sigma(x)]$ describes the self-interactions of the σ field

$$\mathcal{L}_{\rm int}[\sigma(x)] = -\kappa\,M_\sigma\,\sigma^3(x) - \frac{1}{2}\kappa^2\,\sigma^4(x). \quad (1.21)$$

The gap equation, in turn, can be derived in the one-monopole-loop approximation by using only the Lagrangian (1.1)

$$\begin{aligned}&\bar\chi(x)\{2{\rm i}G{\rm Tr}[S_F(0)] - 2{\rm i}GS_F(0) + 2{\rm i}G_1\gamma^\mu S_F(0)\gamma_\mu\}\chi(x)\\ &= \bar\chi(x)\left\{\frac{M}{2\pi^2}J_1(M)\left[-\frac{3}{4}\,G - G_1\right]\right\}\chi(x). \quad (1.22)\end{aligned}$$

Due to the prescription of the NJL model [5] and, correspondingly, the MNJL model [2,3], the result of the calculation of one-loop corrections should be equated to the total mass of the fermion in the superconducting phase. In the MNJL model, this is $-M\,\bar\chi(x)\chi(x)$. Equating the r.h.s. of (1.22) to $-M\,\bar\chi(x)\chi(x)$, we get

$$\begin{aligned}&\bar\chi(x)\left\{\frac{M}{2\pi^2}J_1(M)\left[-\frac{3}{4}\,G - G_1\right]\right\}\chi(x)\\ &= -M\,\bar\chi(x)\chi(x). \quad (1.23)\end{aligned}$$

This leads to the relation

$$\begin{aligned}M &= \left(\frac{3}{4}\,G + G_1\right)\frac{M}{2\pi^2}J_1(M)\\ &= -2\left(\frac{3}{4}\,G + G_1\right)\langle\bar\chi(0)\chi(0)\rangle\,, \quad (1.24)\end{aligned}$$

where we have used (1.18). This is the gap equation also.

Since both gap equations (1.7) and (1.24) describe the same quantity, i.e., the mass of the magnetic monopole field in the superconducting phase, these gap equations should give the same result. Equating these equations, we get the relation $G_1 = G/4$, which reduces the number of input parameters.

2 Magnetic monopole Green functions

The evaluation of the confinement potential accounting for contributions of quantum fluctuations of massive magnetic monopole fields $\chi_M(x)$ and the fields of the collective excitations σ and C_μ is convenient to perform in terms of the generating functional of the magnetic monopole Green functions. The n- point magnetic monopole Green function can be defined as the VEV of the time-ordered product of the massless magnetic monopole densities [2,3,6]:

$$\begin{aligned}&G\,(x_1,\ldots,x_n)\\ &=<0|{\rm T}(\bar\chi(x_1)\varGamma_1\chi(x_1)\ldots\bar\chi(x_n)\,\varGamma_n\chi\,(x_n))|0>_{\rm conn.}\,, (2.1)\end{aligned}$$

where $\varGamma_i (i = 1,\ldots,n)$ are the Dirac matrices. As has been shown in [6], the vacuum expectation value (2.1) can be represented in terms of the vacuum expectation values of the densities of the massive magnetic monopole fields $\chi_M(x)$ coupled to the fields of the collective excitations σ and C_μ:

$$\begin{aligned}&G(x_1,\ldots,x_n)\\ &=<0|{\rm T}(\bar\chi(x_1)\varGamma_1\chi(x_1)\ldots\bar\chi(x_n)\varGamma_n\chi(x_n))|0>_{\rm conn.}\\ &=^{(M)}<0|{\rm T}\Big(\bar\chi_M(x_1)\varGamma_1\chi_M(x_1)\ldots\bar\chi_M(x_n)\varGamma_n\chi_M(x_n)\\ &\times\exp{\rm i}\int{\rm d}^4x\{-g\bar\chi_M(x)\gamma^\nu\chi_M(x)C_\nu(x)\\ &-\kappa\bar\chi_M(x)\chi_M(x)\sigma(x) + \mathcal{L}_{\rm int}[\sigma(x)]\}\Big)|0>^{(M)}_{\rm conn.}\,. \quad (2.2)\end{aligned}$$

Here $|0>^{(M)}$ is the wave function of the nonperturbative vacuum of the MNJL model in the condensed phase, and $|0>$ the wave function of the perturbative vacuum of the noncondensed phase.

The self-interactions $\mathcal{L}_{\rm int}[\sigma(x)]$ provide σ-field loop contributions and can be dropped out in the tree σ-field approximation [2–4]. The tree σ-field approximation can be used by keeping massive magnetic monopoles very heavy, i.e., $M \gg M_C$. This corresponds to the London limit $M_\sigma = 2\,M \gg M_C$ in the dual Higgs model with dual Dirac strings [7–9]. The inequality $M_\sigma \gg M_C$ means also that in the MNJL model, we deal with *dual superconductivity of type* II [4]. In the tree σ-field approximation, the r.h.s. of (2.1) acquires the form

$$\begin{aligned}&G(x_1,\ldots,x_n)\\ &=<0|{\rm T}(\bar\chi(x_1)\varGamma_1\chi(x_1)\ldots\bar\chi(x_n)\varGamma_n\chi(x_n))|0>_{\rm conn.}\\ &=^{(M)}<0|{\rm T}\Big(\bar\chi_M(x_1)\varGamma_1\chi_M(x_1)\ldots\bar\chi_M(x_n)\varGamma_n\chi_M(x_n)\\ &\times\exp{\rm i}\int{\rm d}^4x\Big\{-g\bar\chi_M(x)\gamma^\nu\chi_M(x)C_\nu(x)\\ &-\kappa\bar\chi_M(x)\chi_M(x)\sigma(x)Big\}\Big)|0>^{(M)}_{\rm conn.}\,. \quad (2.3)\end{aligned}$$

For the subsequent investigation, it is convenient to represent the r.h.s. of (2.3) in terms of the generating functional of the monopole Green functions [2–4]

$$G(x_1,\ldots,x_n) = \prod_{i=1}^{n}\frac{\delta}{\delta\eta(x_i)}\varGamma_i\frac{\delta}{\delta\bar\eta(x_i)}Z[\eta,\bar\eta]\Bigg|_{\eta=\bar\eta=0}\,, \quad (2.4)$$

where $\bar\eta(\eta)$ are the external sources of the massive monopole (antimonopole) fields, and $Z[\eta,\bar\eta]$ is the generating functional of the monopole Green functions defined by

$$\begin{aligned}
&Z[\eta,\bar\eta]\\
&=\frac{1}{Z}\int\mathcal{D}\chi_M\mathcal{D}\bar\chi_M\mathcal{D}C_\mu\mathcal{D}\sigma\,\exp\mathrm{i}\int\mathrm{d}^4x\\
&\times\Big[\frac{1}{4}F_{\mu\nu}(x)\,F^{\mu\nu}(x)+\frac{1}{2}M_C^2\,C_\mu(x)\,C^\mu(x)\\
&+\frac{1}{2}\partial_\mu\sigma(x)\,\partial^\mu\sigma(x)-\frac{1}{2}M_\sigma^2\,\sigma^2(x)\\
&+\bar\chi_M(x)(\mathrm{i}\,\gamma^\mu\,\partial_\mu-M-g\,\gamma^\mu\,C_\mu(x)-\kappa\,\sigma(x))\,\chi_M(x)\\
&+\bar\eta(x)\,\chi_M(x)+\bar\chi_M(x)\,\eta(x)+\mathcal{L}_{\text{free quark}}(x)\Big].
\end{aligned}\tag{2.5}$$

The normalization factor Z is defined by the condition $Z[0,0]=1$. Then $\mathcal{L}_{\text{free quark}}(x)$ is the kinetic term for the quark and antiquark

$$\begin{aligned}
\mathcal{L}_{\text{free quark}}(x)=-\sum_{i=q,\bar q}m_i\int\mathrm{d}\tau\left(\frac{\mathrm{d}X_i^\mu(\tau)}{\mathrm{d}\tau}\frac{\mathrm{d}X_i^\nu(\tau)}{\mathrm{d}\tau}g_{\mu\nu}\right)^{1/2}\\
\times\delta^{(4)}(x-X_i(\tau)).
\end{aligned}\tag{2.6}$$

In our consideration, quarks and antiquarks are classical point-like particles with masses $m_q=m_{\bar q}=m$, electric charges $Q_q=-Q_{\bar q}=Q$, and trajectories $X_q^\nu(\tau)$ and $X_{\bar q}^\nu(\tau)$, respectively. The field strength $F^{\mu\nu}(x)$ is defined [2–4] as $F^{\mu\nu}(x)=\mathcal{E}^{\mu\nu}(x)-{}^*dC^{\mu\nu}(x)$, where $dC^{\mu\nu}(x)=\partial^\mu C^\nu(x)-\partial^\nu C^\mu(x)$, and ${}^*dC^{\mu\nu}(x)$ is the dual version, i.e., ${}^*dC^{\mu\nu}(x)=1/2\varepsilon^{\mu\nu\alpha\beta}dC_{\alpha\beta}(x)$ ($\varepsilon^{0123}=1$). The dual chrom electric field strength $\mathcal{E}^{\mu\nu}(x)$, induced by a dual Dirac string, is defined following [2–4], as

$$\begin{aligned}
\mathcal{E}^{\mu\nu}(x)=Q\iint\mathrm{d}\tau\mathrm{d}\sigma\left(\frac{\partial X^\mu}{\partial\tau}\frac{\partial X^\nu}{\partial\sigma}-\frac{\partial X^\nu}{\partial\tau}\frac{\partial X^\mu}{\partial\sigma}\right)\\
\times\delta^{(4)}(x-X),
\end{aligned}\tag{2.7}$$

where $X^\mu=X^\mu(\tau,\sigma)$ represents the position of a point on the world sheet swept by the string. The sheet is parameterized by internal coordinates $-\infty<\tau<\infty$ and $0\le\sigma\le\pi$, so that $X^\mu(\tau,0)=X^\mu_{-Q}(\tau)$ and $X^\mu(\tau,\pi)=X^\mu_Q(\tau)$ represent the world lines of an antiquark and a quark [2–4,6,7]. For the definition (2.7), the tensor field $\mathcal{E}^{\mu\nu}(x)$ satisfies identically the equation of motion, $\partial_\mu F^{\mu\nu}(x)=J^\nu(x)$. The electric quark current $J^\nu(x)$ is defined as

$$J^\nu(x)=\sum_{i=q,\bar q}Q_i\int\mathrm{d}\tau\frac{\mathrm{d}X_i^\nu(\tau)}{\mathrm{d}\tau}\delta^{(4)}(x-X_i(\tau)).\tag{2.8}$$

Hence, the inclusion of a dual Dirac string in terms of $\mathcal{E}^{\mu\nu}(x)$ defined by (2.7) satisfies completely the dual electric Gauss law of Dirac′s extension of Maxwell′s electrodynamics.

The ground state of the massive dual-vector field $C_\mu(x)$ coupled to a dual Dirac string acquires the shape of the Abrikosov flux line [2–4,7,8]

$$C^\nu[\mathcal{E}(x)]=-\int\mathrm{d}^4x'\,\Delta(x-x')\,\partial_\mu{}^*\mathcal{E}^{\mu\nu}(x'),\tag{2.9}$$

where $\Delta(x-x')$ is the Green function

$$\Delta(x-x')=\int\frac{\mathrm{d}^4k}{(2\pi)^4}\,\frac{\mathrm{e}^{-\mathrm{i}k\cdot(x-x')}}{M_C^2-k^2-\mathrm{i}0}.\tag{2.10}$$

Integrating out the dual-vector field fluctuations $c_\mu(x)$ around the shape of the Abrikosov flux line, $C_\mu(x)=C_\mu[\mathcal{E}(x)]+c_\mu(x)$, and the scalar σ field [4], we obtain the generating functional of the monopole Green functions in the following form:

$$\begin{aligned}
Z[\eta,\bar\eta]=\frac{1}{Z}\int\mathcal{D}\chi_M\mathcal{D}\bar\chi_M\,\exp\mathrm{i}\int\mathrm{d}^4x\\
\Big[\mathcal{L}_{\text{eff}}\{\bar\chi_M(x),\chi_M(x),C^\nu[\mathcal{E}(x)]\}\\
+\bar\chi_M(x)(\mathrm{i}\,\gamma^\mu\,\partial_\mu-M\\
-g\,\gamma^\mu\,C_\mu[\mathcal{E}(x)])\,\chi_M(x)+\bar\eta(x)\,\chi_M(x)\\
+\bar\chi_M(x)\,\eta(x)+\mathcal{L}_{\text{free quark}}(x)\Big],
\end{aligned}\tag{2.11}$$

where $\mathcal{L}_{\text{eff}}\{\bar\chi_M(x),\chi_M(x),C^\nu[\mathcal{E}(x)]\}$ reads

$$\begin{aligned}
&\mathcal{L}_{\text{eff}}\{\bar\chi_M(x),\chi_M(x),C^\nu[\mathcal{E}(x)]\}\\
&=\mathcal{L}_{\text{string}}\{C^\nu[\mathcal{E}(x)]\}-\frac{g^2}{2M_C^2}[\bar\chi_M(x)\gamma_\mu\chi_M(x)]\\
&\times[\bar\chi_M(x)\gamma^\mu\chi_M(x)]+\frac{\kappa^2}{2M_\sigma^2}[\bar\chi_M(x)\chi_M(x)]^2.
\end{aligned}\tag{2.12}$$

The Lagrangian of the dual Dirac string $\mathcal{L}_{\text{string}}\{C^\nu[\mathcal{E}(x)]\}$ is defined as [3,4,8]

$$\begin{aligned}
&\int\mathrm{d}^4x\mathcal{L}_{\text{string}}\{C^\nu[\mathcal{E}(x)]\}\\
&=\frac{1}{4}M_C^2\iint\mathrm{d}^4x\mathrm{d}^4y\mathcal{E}_{\mu\alpha}(x)\Delta^\alpha_\nu(x-y,M_C)\mathcal{E}^{\mu\nu}(y),
\end{aligned}\tag{2.13}$$

where $\Delta^\alpha_\nu(x-y,M_C)=(g^\alpha_\nu+2\partial^\alpha\partial_\nu/M_C^2)\Delta(x-y;M_C)$.

The effective Lagrangian (2.12) integrated over the massive monopole fields $\bar\chi_M(x)$ and $\chi_M(x)$ defines the string energy, i.e., the interquark potential, as a functional of the string shape. Thus, for the evaluation of the interquark potential accounting for quantum fluctuations of the massive magnetic monopole fields $\chi_M(x)$ and the fields of the collective excitations σ and C_μ, we have to average only the effective Lagrangian (2.12) over the massive monopole fields $\bar\chi_M(x)$ and $\chi_M(x)$.

3 Confinement potential

The interquark confinement potential W or the string energy we define as [2–4,6,7]

$$W=-\int\mathrm{d}^3x\,\langle\mathcal{L}_{\text{eff}}\{\bar\chi_M(x),\chi_M(x),C^\nu[\mathcal{E}(x)]\}\rangle\,,\tag{3.1}$$

where the brackets assume the integration over the massive magnetic monopole fields

$$\langle\mathcal{L}_{\text{eff}}\{\bar\chi_M(x),\chi_M(x),C^\nu[\mathcal{E}(x)]\}\rangle$$

$$= \frac{1}{Z}\int \mathcal{D}\chi_M \mathcal{D}\bar{\chi}_M \mathcal{L}_{\rm eff}\{\bar{\chi}_M(x),\chi_M(x),C^{\nu}[\mathcal{E}(x)]\}$$
$$\times \exp{\rm i}\int {\rm d}^4x\,\Big[\bar{\chi}_M(x)({\rm i}\gamma^{\mu}\partial_{\mu} - M - g\gamma^{\mu}C_{\mu}[\mathcal{E}(x)])$$
$$\times\chi_M(x) + \bar{\eta}(x)\chi_M(x) + \bar{\chi}_M(x)\eta(x)\Big]. \qquad (3.2)$$

This equation can be reduced to the more simple form

$$W = -\int {\rm d}^3x\,\mathcal{L}_{\rm string}\{C^{\nu}[\mathcal{E}(x)]\} + \int {\rm d}^3x\,{}^{(M)}<0|{\rm T}$$
$$\times\Big(\Big(-\frac{g^2}{2M_C^2}[\bar{\chi}_M(x)\gamma_{\mu}\chi_M(x)]\,[\bar{\chi}_M(x)\gamma^{\mu}\chi_M(x)]$$
$$+\frac{\kappa^2}{2M_{\sigma}^2}[\bar{\chi}_M(x)\chi_M(x)]^2\Big) \qquad (3.3)$$
$$\times\exp -{\rm i}g\int {\rm d}^4y\,\bar{\chi}_M(y)\,\gamma^{\mu}\,C_{\mu}[\mathcal{E}(y)]\,\chi_M(y)\Big)|0>^{(M)}.$$

The interaction caused by the integration over the σ-field fluctuations gives a trivial constant contribution to the energy of the string [4] and can be dropped out. In the momentum representation of the vacuum expectation values, the energy of the string is then defined by [4]:

$$W = -\int {\rm d}^3x\,\mathcal{L}_{\rm string}\{C^{\nu}[\mathcal{E}(x)]\} - \int {\rm d}^3x\,\frac{g^2}{2M_C^2}$$
$$\times\int\frac{d^4k_1}{(2\pi)^4{\rm i}}\,{\rm Tr}\Big\{\frac{1}{M-\hat{k}_1+g\hat{C}[\mathcal{E}(x)]}\gamma^{\mu}\Big\}$$
$$\times\int\frac{{\rm d}^4k_2}{(2\pi)^4{\rm i}}\,{\rm Tr}\Big\{\gamma_{\mu}\frac{1}{M-\hat{k}_2+g\hat{C}[\mathcal{E}(x)]}\Big\}. \qquad (3.4)$$

The momentum integrals have been calculated in [4]. This yields the energy of the string:

$$W = -\int {\rm d}^3x\,\mathcal{L}_{\rm string}\{C^{\nu}[\mathcal{E}(x)]\}$$
$$-\frac{1}{2}\,\frac{1}{M_C^2}\left(\frac{g^2}{8\pi^2}[J_1(M)+M^2J_2(M)]\right)^2$$
$$\times\int {\rm d}^3x\,C_{\mu}[\mathcal{E}(x)]\,C^{\mu}[\mathcal{E}(x)]. \qquad (3.5)$$

By using (1.7), (1.8), and (1.12), and the relations $G_1 = G/4$ and $M_{\sigma} = 2\,M$, we bring up the coefficient of the second term to the form

$$-\frac{g^2}{8\pi^2}[J_1(M)+M^2J_2(M)]$$
$$= M_C^2 - \frac{g^2}{2G_1} = M_C^2 + 8\,g^2\frac{\langle\bar{\chi}\chi\rangle}{M_{\sigma}}. \qquad (3.6)$$

Thus, the energy of the string containing quantum fluctuations of the scalar and dual-vector fields around the shape of the Abrikosov flux line is given by

$$W = -\int {\rm d}^3x\mathcal{L}_{\rm string}\{C^{\nu}[\mathcal{E}(x)]\} - \frac{1}{2}\,M_C^2\left(1+\frac{8g^2}{M_C^2}\frac{\langle\bar{\chi}\chi\rangle}{M_{\sigma}}\right)^2$$
$$\times\int {\rm d}^3x\,C_{\mu}[\mathcal{E}(x)]\,C^{\mu}[\mathcal{E}(x)]. \qquad (3.7)$$

We perform the computation of the r.h.s. of (3.5) for the static straight string of the length L directed along the z axis. In this case, the electric field strength $\mathcal{E}_{\mu\nu}(x)$ does not depend on time, and is given by [8]

$$\vec{\mathcal{E}}(\vec{x}\,) = \vec{e}_z\,Q\,\delta(x)\,\delta(y)$$
$$\times\left[\theta\left(z-\frac{1}{2}\,L\right)-\theta\left(z+\frac{1}{2}\,L\right)\right], \qquad (3.8)$$

where a quark and an antiquark are placed at $\vec{X}_Q = (0,0,\frac{1}{2}\,L)$ and $\vec{X}_{-Q} = (0,0,-1/2\,L)$. The unit vector $\vec{e}_z$ is directed along the z axis, and $\theta(z)$ is the step function. The field strength (3.8) induces the dual-vector potential

$$\Big\langle\vec{C}(\vec{x})\Big\rangle \qquad (3.9)$$
$$= -\,{\rm i}\,Q\int\frac{{\rm d}^3k}{4\,\pi^3}\,\frac{\vec{k}\times\vec{e}_z}{k_z}\,\frac{1}{M_C^2+\vec{k}^{\,2}}\sin\left(\frac{k_zL}{2}\right){\rm e}^{{\rm i}\,\vec{k}\,\cdot\,\vec{x}}.$$

For the static straight string, the term in Eq. (3.7) reads

$$-\int {\rm d}^3x\mathcal{L}_{\rm string}\{C^{\nu}[\mathcal{E}(x)]\}$$
$$= -\frac{1}{4}\,M_C^2\int {\rm d}^3x\int {\rm d}^3x'\int_{-\infty}^{\infty}{\rm d}x_0'\Big[\mathcal{E}_{0i}(\vec{x}\,)\Big(g^i_j+\frac{2}{M_C^2}$$
$$\times\frac{\partial^2}{\partial x_i\partial x^j}\Big)\,\Delta(x_0-x_0',\vec{x}-\vec{x}\,',M_C)\,\mathcal{E}^{0j}(\vec{x}\,')+\mathcal{E}_{i0}(\vec{x})$$
$$\times\Big(g^0_0+\frac{2}{M_C^2}\frac{\partial^2}{\partial x_0\partial x^0}\Big)\Delta(x_0-x_0',\vec{x}-\vec{x}',M_C)\mathcal{E}^{i0}(\vec{x}')\Big]$$
$$= \frac{1}{2}\,M_C^2\int {\rm d}^3x\int {\rm d}^3x'\,\vec{\mathcal{E}}(\vec{x}\,)\cdot\vec{\mathcal{E}}(\vec{x}\,')\left(1-\frac{1}{M_C^2}\frac{\partial^2}{\partial z^2}\right)$$
$$\times\Delta(\vec{x}-\vec{x}\,',M_C)$$
$$= \frac{1}{2}\,Q^2M_C^2\int_{-L/2}^{L/2}{\rm d}z\int_{-L/2}^{L/2}{\rm d}z'\int_{-\infty}^{\infty}{\rm d}k_z\left(1+\frac{k_z^2}{M_C^2}\right)$$
$$\times\int\frac{{\rm d}^2k_{\perp}}{(2\pi)^3}\,\frac{{\rm e}^{{\rm i}k_z(z\,-\,z'\,)}}{M_C^2+\vec{k}_{\perp}^{\,2}+k_z^2}$$
$$= \frac{Q^2M_C^2}{4\pi^3}\int_{-\infty}^{\infty}\frac{{\rm d}k_z}{k_z^2}\,\sin^2\left(\frac{k_zL}{2}\right)\left(1+\frac{k_z^2}{M_C^2}\right)$$
$$\times\int\frac{{\rm d}^2k_{\perp}}{M_C^2+\vec{k}_{\perp}^{\,2}+k_z^2}$$
$$= \frac{Q^2M_C^2}{4\pi^2}\int_{-\infty}^{\infty}\frac{{\rm d}k_z}{k_z^2}\,\sin^2\left(\frac{k_zL}{2}\right)\left(1+\frac{k_z^2}{M_C^2}\right)$$
$$\times\int_0^{\Lambda_{\perp}^2}\frac{{\rm d}k_{\perp}^2}{M_C^2+k_{\perp}^2+k_z^2}, \qquad (3.10)$$

where $\Lambda_\perp$ is the cutoff in the plane perpendicular to the world sheet swept by the string [2–4,7,8]. We identify $\Lambda_\perp$ with the mass of the scalar field, i.e., $\Lambda_\perp = M_\sigma = 2M$ [2–4,7,8].

For a sufficiently long string, we can integrate over $k_\perp^2$ and get

$$-\int \mathrm{d}^3x \mathcal{L}_{\mathrm{string}}\{C^\nu[\mathcal{E}(x)]\} = \frac{Q^2M_C^2}{4\pi^2}\int\limits_{-\infty}^{\infty}\frac{\mathrm{d}k_z}{k_z^2}\sin^2\left(\frac{k_zL}{2}\right)\left(1+\frac{k_z^2}{M_C^2}\right) \times\left[\ln\left(1+\frac{M_\sigma^2}{M_C^2}\right)-\ln\left(1+\frac{k_z^2}{M_C^2}\right)\right], \quad (3.11)$$

where we have neglected k_z relative to $\Lambda_\perp$. Dropping the infinite constant contributions independent of L, we obtain [8]:

$$-\int \mathrm{d}^3x\, \mathcal{L}_{\mathrm{string}}\{C^\nu[\mathcal{E}(x)]\} = L\frac{Q^2M_C^2}{8\pi}\left[\ln\left(1+\frac{M_\sigma^2}{M_C^2}\right)+2\,E_1(M_CL) -\frac{2}{M_CL}\left(1-\mathrm{e}^{-M_CL}\right)\right]-\frac{Q^2}{4\pi}\frac{\mathrm{e}^{-M_CL}}{L}, \quad (3.12)$$

where $E_1(M_CL)$ is the exponential integral function. For the calculation of the integral over k_z, we have used the auxiliary integral

$$\int\limits_{-\infty}^{\infty}\mathrm{d}x\,\frac{\sin^2 x}{x^2}\ln\left(\alpha^2+\frac{x^2}{a^2}\right) = 2\pi\ln\alpha+\frac{\pi}{a\alpha}\left(1-\mathrm{e}^{-2a\alpha}\right)-2\pi\,E_1(2a\alpha), \quad (3.13)$$

where in (3.12), we have set $\alpha = 1$ and $a = M_CL/2$.

The first term proportional to L gives the string tension σ_0 calculated in the tree approximation [7]:

$$\sigma_0 = \frac{Q^2M_C^2}{8\pi}\ln\left(1+\frac{M_\sigma^2}{M_C^2}\right). \quad (3.14)$$

The last term in (3.7) induced by the quantum fluctuations of the dual-vector field C_μ around the shape of the Abrikosov flux line can be reduced to the form [8]:

$$-\frac{1}{2}M_C^2\left(1+\frac{8g^2}{M_C^2}\frac{\langle\bar{\chi}\chi\rangle}{M_\sigma}\right)^2\int \mathrm{d}^3x\,C_\mu[\mathcal{E}(x)]\,C^\mu[\mathcal{E}(x)] = \frac{Q^2M_C^2}{4\pi^2}\left(1+\frac{8g^2}{M_C^2}\frac{\langle\bar{\chi}\chi\rangle}{M_\sigma}\right)^2\int\limits_{-\infty}^{\infty}\frac{\mathrm{d}k_z}{k_z^2}\sin^2\left(\frac{k_zL}{2}\right) \times\left[\ln\left(1+\frac{M_\sigma^2}{M_C^2}\right)-\ln\left(1+\frac{k_z^2}{M_C^2}\right)\right]$$

$$= L\,\frac{Q^2M_C^2}{8\pi}\left(1+\frac{8g^2}{M_C^2}\frac{\langle\bar{\chi}\chi\rangle}{M_\sigma}\right)^2\left[\ln\left(1+\frac{M_\sigma^2}{M_C^2}\right) +2\,E_1(M_CL)-\frac{2}{M_CL}\left(1-\mathrm{e}^{-M_CL}\right)\right]. \quad (3.15)$$

Collecting the pieces together, we obtain the energy of the dual Dirac string, the interquark potential, as a function of the length of the string L:

$$W = L\,\frac{Q^2M_C^2}{4\pi}\left(1+\frac{8g^2}{M_C^2}\frac{\langle\bar{\chi}\chi\rangle}{M_\sigma}+\frac{32g^4}{M_C^4}\frac{\langle\bar{\chi}\chi\rangle^2}{M_\sigma^2}\right) \times\left[\ln\left(1+\frac{M_\sigma^2}{M_C^2}\right)+2\,E_1(M_CL) -\frac{2}{M_CL}\left(1-\mathrm{e}^{-M_CL}\right)\right]-\frac{Q^2}{4\pi}\frac{\mathrm{e}^{-M_CL}}{L}. \quad (3.16)$$

The term proportional to L describes a linearly rising interquark potential leading to the quark confinement and gives the expression for the string tension

$$\sigma = \frac{Q^2M_C^2}{4\pi}\left(1+\frac{8g^2}{M_C^2}\frac{\langle\bar{\chi}\chi\rangle}{M_\sigma}+\frac{32g^4}{M_C^4}\frac{\langle\bar{\chi}\chi\rangle^2}{M_\sigma^2}\right) \times\ln\left(1+\frac{M_\sigma^2}{M_C^2}\right). \quad (3.17)$$

The last term in (3.15) are the Yukawa potential.

Matching the string tension (3.17) with the string tension σ_0 calculated in the tree approximation (3.15), we accentuate a tangible contribution of quantum fluctuations of the dual-vector field C_μ around the shape of the Abrikosov flux line. This agrees with the result obtained in [8] in the dual Higgs model.

4 String shape fluctuations

The string shape fluctuations we usually define as $X^\mu \to X^\mu + \eta^\mu(X)$ [10,9], where $\eta^\mu(X)$ describes fluctuations around the fixed surface S swept by the shape Γ and obeys the constraint $\eta^\mu(X)|_{\partial S} = 0$ [10,9] at the boundary ∂S of the surface S. We perform the integration over the η field around the shape of the static straight string, with the length L tracing out the rectangular surface S with the time-side T [10,9]. Allowing only fluctuations in the plane perpendicular to the string world sheet and setting $\eta_t(t,z) = \eta_z(t,z) = 0$ [10,9], we arrive at the fluctuation action $\delta\,\mathcal{S}_{\mathrm{N}}[\eta_x,\eta_y]$ [9,4]

$$\delta\,\mathcal{S}_{\mathrm{N}}[\eta_x,\eta_y] = -\frac{3Q^2\Lambda_\perp^2}{32\pi}\int\limits_{-T/2}^{T/2}\mathrm{d}t\int\limits_{-L/2}^{L/2}\mathrm{d}z[\eta_x(t,z) \times(-\Delta)\,\eta_x(t,z)+(x\leftrightarrow y)], \quad (4.1)$$

coming from the term $\int \mathrm{d}^4x\, \mathcal{L}_{\text{string}}\{C^{\nu}[\mathcal{E}(x)]\}$ defined by (2.13), where Δ is the Laplace operator in two-dimensional space-time:

$$\Delta = -\frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial z^2}. \tag{4.2}$$

The term $\int \mathrm{d}^4x\, C_\mu[\mathcal{E}(x)]\, C^\mu[\mathcal{E}(x)]$ in (3.5), induced by the quantum fluctuations of the dual-vector C_μ and scalar σ fields around the shape of the Abrikosov flux line, does not contribute to the fluctuation action for the case of the static straight string. In order to show this we use the expression obtained in [4]:

$$\begin{aligned}
&\delta C_\mu[\mathcal{E}(x)]\, C^\mu[\mathcal{E}(x)] \\
&= Q^2 \iint \frac{\mathrm{d}^3k}{4\pi^3}\frac{\mathrm{d}^3q}{4\pi^3}\frac{k_xq_x + k_yq_y}{k_zq_z}\sin\left(\frac{k_zL}{2}\right)\sin\left(\frac{q_zL}{2}\right) \\
&\quad\times \frac{1}{M_C^2+\vec{k}^{\,2}}\frac{1}{M_C^2+\vec{q}^{\,2}}\,\mathrm{e}^{\mathrm{i}\,(\vec{k}+\vec{q})\cdot\vec{x}} \\
&\quad\times\left(\mathrm{e}^{\mathrm{i}\,[(k_x+q_x)\eta_x(t,z)+(k_y+q_y)\eta_y(t,z)]}-1\right).
\end{aligned} \tag{4.3}$$

The contribution to the fluctuation action is given by

$$\begin{aligned}
&\int \mathrm{d}^4x\, \delta C_\mu[\mathcal{E}(x)]\, C^\mu[\mathcal{E}(x)] \\
&= Q^2 \int d^4x \iint \frac{\mathrm{d}^3k}{4\pi^3}\frac{\mathrm{d}^3q}{4\pi^3}\frac{k_xq_x + k_yq_y}{k_zq_z}\sin\left(\frac{k_zL}{2}\right) \\
&\quad\times\sin\left(\frac{q_zL}{2}\right)\frac{1}{M_C^2+\vec{k}^{\,2}}\,\frac{1}{M_C^2+\vec{q}^{\,2}}\,\mathrm{e}^{\mathrm{i}\,(\vec{k}+\vec{q})\cdot\vec{x}} \\
&\quad\times\left(\mathrm{e}^{\mathrm{i}\,[(k_x+q_x)\eta_x(t,z)+(k_y+q_y)\eta_y(t,z)]}-1\right).
\end{aligned} \tag{4.4}$$

Integrating over x and y, we get

$$\begin{aligned}
&\int \mathrm{d}^4x\, \delta C_\mu[\mathcal{E}(x)]\, C^\mu[\mathcal{E}(x)] \\
&= Q^2 \int\limits_{-T/2}^{T/2}\mathrm{d}t \int\limits_{-L/2}^{L/2}\mathrm{d}z \iint \frac{\mathrm{d}^3k}{2\pi^2}\frac{\mathrm{d}^3q}{2\pi^2}\frac{k_xq_x + k_yq_y}{k_zq_z} \\
&\quad\times\sin\left(\frac{k_zL}{2}\right)\sin\left(\frac{q_zL}{2}\right)\frac{1}{M_C^2+\vec{k}^{\,2}}\,\frac{1}{M_C^2+\vec{q}^{\,2}} \\
&\quad\times e^{i\,(k_0+q_0)t - i(k_z+q_z)z}\,\delta(k_x+q_x)\,\delta(k_y+q_y) \\
&\quad\times\left(e^{i\,[(k_x+q_x)\eta_x(t,z)+(k_y+q_y)\eta_y(t,z)]}-1\right) \\
&= 0.
\end{aligned} \tag{4.5}$$

Thus, (4.1) defines completely the fluctuation action induced by string shape fluctuations around a static straight string with length L. As has been shown in [9], the fluctuation action (4.1) gives a Coulomb-like universal contribution [10] to the energy of the string:

$$W_{\text{string}-\text{shape}} = -\frac{\alpha_{\text{string}}}{L}, \tag{4.6}$$

where $\alpha_{\text{string}} = \pi/12$ and $\alpha_{\text{string}} = \pi/3$ for opened and closed strings, respectively.

5 Conclusion

We have shown that in the MNJL model with dual Dirac strings, the quantum fluctuations of a dual-vector field C_μ and a scalar field σ around the shape of the Abrikosov flux line give the interquark confinement potential in the following form:

$$\begin{aligned}
W_{\text{tot}} &= L\,\frac{Q^2M_C^2}{4\pi}\left(1+\frac{8g^2}{M_C^2}\frac{\langle\bar{\chi}\chi\rangle}{M_\sigma}+\frac{32g^4}{M_C^4}\frac{\langle\bar{\chi}\chi\rangle^2}{M_\sigma^2}\right) \\
&\quad\times\left[\ln\left(1+\frac{M_\sigma^2}{M_C^2}\right)+2\,E_1(M_CL)-\frac{2}{M_CL}\right. \\
&\quad\left.\times\left(1-\mathrm{e}^{-M_CL}\right)\right]-\frac{Q^2}{4\pi}\frac{\mathrm{e}^{-M_CL}}{L}-\frac{\alpha_{\text{string}}}{L},
\end{aligned} \tag{5.1}$$

where $\alpha_{\text{string}} = \pi/12$ and $\alpha_{\text{string}} = \pi/3$ for opened and closed strings, respectively. This interquark potential resembles the result obtained in the dual Higgs model with dual Dirac strings [7–9]. Unlike in the dual Higgs model with dual Dirac strings [7], the mass of a dual-vector field M_C is not proportional to the order parameter $\langle\bar{\chi}\chi\rangle$ and does not vanish in the limit $\langle\bar{\chi}\chi\rangle \to 0$. This is seen from the mass formula [4],

$$M_\sigma\,(8\,M_C^2+3\,M_\sigma^2) = -56g^2\,\langle\bar{\chi}\chi\rangle\,, \tag{5.2}$$

which can be derived from (1.2), and the gap equation (1.7). Thus, in the MNJL model the dual-vector field does not need a Goldstone boson as a longitudinal component. This distinguishes the transition to the nonperturbative superconducting phase in the MNJL from that in the dual Higgs model. Indeed, in the MNJL model, this transition does not accompany the appearance of Goldstone bosons. The former is rather natural, since the starting U(1) magnetic symmetry in the MNJL model is global and unbroken in the nonperturbative superconducting phase. Recall that in the dual Higgs model, the magnetic U(1) symmetry is local and becomes spontaneously broken in the superconducting phase.

Due to the independence of the mass of the dual-vector field on the monopole condensate, the string tension σ_0 calculated in the tree approximation does not depend on the monopole condensate either. The mass of the Higgs field M_σ replaced the cutoff $\Lambda_\perp$, i.e., $\Lambda_\perp = M_\sigma$. The dependence on the magnetic monopole condensate appears by virtue of the contributions of the quantum field fluctuations of the dual-vector C_μ and the scalar σ fields around the shape of the Abrikosov flux line.

In the MNJL model, the contributions of quantum fluctuations to the string tension are dominant in comparison with the contribution calculated at the classical level. In order to make this much more transparent, one uses (5.2) and replaces in the expression for the string tension (3.17) the magnetic monopole condensate $\langle\bar{\chi}\chi\rangle$ by the masses of the σ and C_μ fields:

$$\sigma = \left(\frac{50}{49}+\frac{6}{49}\,\xi^2+\frac{9}{49}\,\xi^4\right)\sigma_0, \tag{5.3}$$

where σ_0 is the string tension calculated at the classical level (3.14); then the value of the parameter $\xi = M_\sigma/M_C$ specifies the type of superconductivity realized in the condensed phase. Indeed, if $\xi \geq \sqrt{2}$, the nonperturbative vacuum behaves like a *dual superconductor of type II*; otherwise, at $\xi < \sqrt{2}$, it is a *dual superconductor of type I* [11]. For $\xi = \sqrt{2}$, we get $\sigma = 2\sigma_0$. However, since in the London limit, $\xi \gg \sqrt{2}$, we predict $\sigma \gg 2\sigma_0$. This confirms the dominant role of quantum fluctuations of quantum fields in the condensed phase for the formation of the confinement potential.

This result supports the result obtained in the dual Higgs model with dual Dirac strings [8]. However, in the MNJL model, such a dominance is displayed much more distinctly. Thus, for the consistent investigation of the superconducting mechanism of the quark confinement within the dynamics of magnetic monopoles and dual Dirac strings, one cannot deal with a classical level only; the contributions of quantum fluctuations of the quantum fields in the condensed phase should be taken into account.

Then, it is shown that in the MNJL model, as well as in the dual Higgs model with dual Dirac strings, the string shape fluctuations in comparison with the quantum fluctuations of the quantum fields in the condensed phase do not influence the string tension, and induce only a Coulomb-like universal contribution calculated earlier for opened strings by Lüscher, et al. [10] and for closed strings by Faber, et al. [9].

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